

**OCR**

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**A Level Mathematics B (MEI)****H640/03 Pure Mathematics and Comprehension**

Sample Question Paper

Version 2

**Date – Morning/Afternoon**

Time allowed: 2 hours

**You must have:**

- Printed Answer Booklet
- the Insert

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

## Formulae A Level Mathematics B (MEI) (H640)

### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

### Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1}b + {}^n C_2 a^{n-2}b^2 + \dots + {}^n C_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

### Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

### Integration

$$\int \frac{f(x)}{f'(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

### Small Angle Approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

**Sample Variance**

$$s^2 = \frac{1}{n-1} S_{xx} \quad \text{where} \quad S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

**The Binomial Distribution**

If  $X \sim B(n, p)$  then  $P(X = r) = {}^n C_r p^r q^{n-r}$  where  $q = 1 - p$

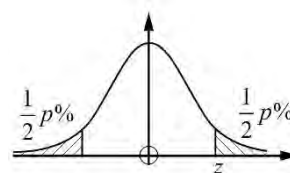
Mean of  $X$  is  $np$

**Hypothesis testing for the mean of a Normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

$p$	10	5	2	1
$z$	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2} at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2} \mathbf{a}t^2$$

Answer **all** the questions

**Section A** (60 marks)

1 Express  $\frac{2}{x-1} + \frac{5}{2x+1}$  as a single fraction. [2]

2 Find the first four terms of the binomial expansion of  $(1-2x)^{\frac{1}{2}}$ .

State the set of values of  $x$  for which the expansion is valid. [4]

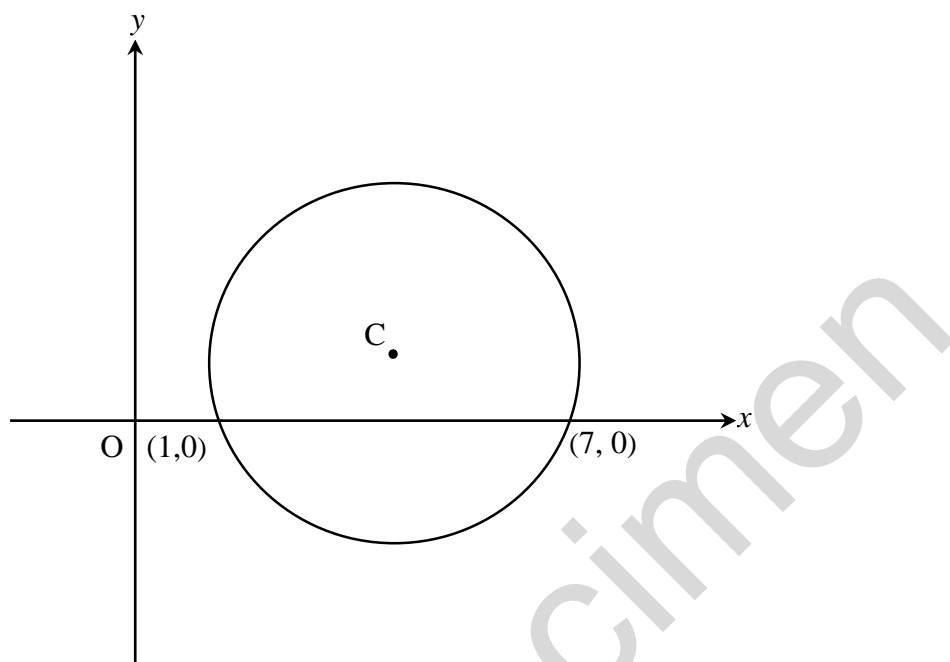
3 Show that points A (1, 4, 9), B (0, 11, 17) and C (3, -10, -7) are collinear. [4]

4 Show that  $\sum_{r=1}^4 \ln \frac{r}{r+1} = -\ln 5$ . [3]

5 In this question you must show detailed reasoning.

**Fig. 5** shows the circle with equation  $(x-4)^2 + (y-1)^2 = 10$ .

The points  $(1, 0)$  and  $(7, 0)$  lie on the circle. The point  $C$  is the centre of the circle.



**Fig. 5**

Find the area of the part of the circle below the  $x$ -axis.

[5]

- 6 Fig. 6 shows the curve with equation  $y = x^4 - 6x^2 + 4x + 5$ .

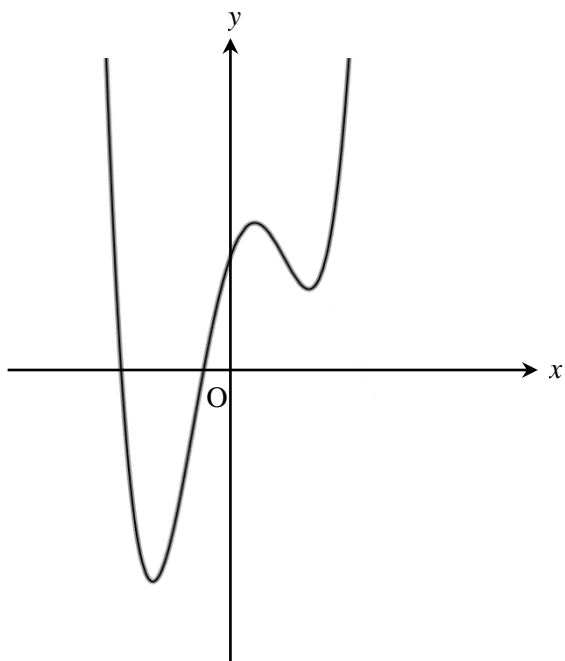


Fig. 6

Find the coordinates of the points of inflection.

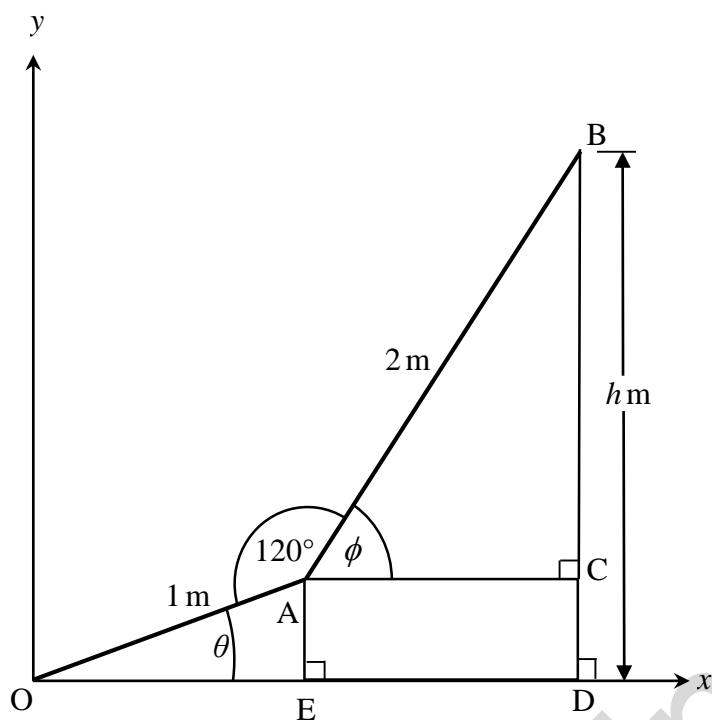
[5]

- 7 By finding a counter example, disprove the following statement.

If  $p$  and  $q$  are non-zero real numbers with  $p < q$ , then  $\frac{1}{p} > \frac{1}{q}$ .

[2]

- 8 In **Fig. 8**, OAB is a thin bent rod, with  $OA = 1$  m,  $AB = 2$  m and angle  $OAB = 120^\circ$ . Angles  $\theta$ ,  $\phi$  and  $h$  are as shown in **Fig. 8**.



**Fig. 8**

- (a) Show that  $h = \sin \theta + 2 \sin(\theta + 60^\circ)$ . [3]

The rod is free to rotate about the origin so that  $\theta$  and  $\phi$  vary. You may assume that the result for  $h$  in part (a) holds for all values of  $\theta$ .

- (b) Find an angle  $\theta$  for which  $h = 0$ . [5]

- 9 (a) Express  $\cos \theta + 2 \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $0 < \alpha < \frac{1}{2} \pi$  and  $R$  is positive and given in exact form. [4]

The function  $f(\theta)$  is defined by  $f(\theta) = \frac{1}{(k + \cos \theta + 2 \sin \theta)}$ ,  $0 \leq \theta \leq 2\pi$ ,  $k$  is a constant.

- (b) The maximum value of  $f(\theta)$  is  $\frac{(3 + \sqrt{5})}{4}$ .  
Find the value of  $k$ . [3]

10 The function  $f(x)$  is defined by  $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$ .

- (a) Show that  $x = -1$  is a root of  $f(x) = 0$ . [1]
- (b) Show that another root of  $f(x) = 0$  lies between  $x = 1$  and  $x = 2$ . [2]
- (c) Show that  $f(x) = (x + 1)g(x)$ , where  $g(x) = x^3 + ax + b$  and  $a$  and  $b$  are integers to be determined. [3]
- (d) Without further calculation, explain why  $g(x) = 0$  has a root between  $x = 1$  and  $x = 2$ . [1]
- (e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of  $g(x) = 0$  may be written

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}.$$

Determine the root of  $g(x) = 0$  which lies between  $x = 1$  and  $x = 2$  correct to 4 significant figures. [3]



**11** The curve  $y = f(x)$  is defined by the function  $f(x) = e^{-x} \sin x$  with domain  $0 \leq x \leq 4\pi$ .

**(a) (i)** Show that the  $x$ -coordinates of the stationary points of the curve  $y = f(x)$ , when arranged in increasing order, form an arithmetic sequence.

**(ii)** Show that the corresponding  $y$ -coordinates form a geometric sequence. **[9]**

**(b)** Would the result still hold with a larger domain? Give reasons for your answer. **[1]**

Specimen

Answer **all** the questions

**Section B** (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

**12** Explain why the smaller regular hexagon in **Fig. C1** has perimeter 6. [1]

**13** Show that the larger regular hexagon in **Fig. C1** has perimeter  $4\sqrt{3}$ . [3]

**14** Show that the two values of  $b$  given on line 36 are equivalent. [3]

Specimen

- 15 Fig. 15 shows a unit circle and the escribed regular polygon with 12 edges.

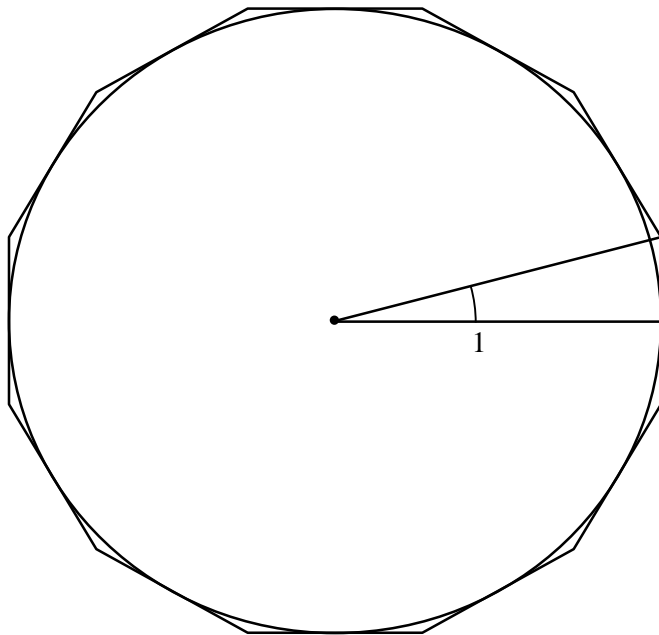


Fig. 15

- (a) Show that the perimeter of the polygon is  $24 \tan 15^\circ$ . [2]
- (b) Using the formula for  $\tan(\theta - \phi)$  show that the perimeter of the polygon is  $48 - 24\sqrt{3}$ . [3]
- 16 On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for  $\pi$ , and the escribed regular polygon with 12 edges gives an upper bound for  $\pi$ .

Calculate the values of these bounds for  $\pi$ , giving your answers:

- (i) in surd form
- (ii) correct to 2 decimal places. [3]

**END OF QUESTION PAPER**

Specimen

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